King Saud University: Mathematics Department Math-254
First Semester 1445 H Final Examination Solution
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————————— **I.D. No.** ——————————

Name of the Teacher:———————————— **Section No.** ——————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15
a,b,c,d					

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.(Math)

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	c	b	a	b	c	a	c	b	b	a	c	b	c	b	a

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.(MAth)

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	b	a	c	c	a	b	b	a	c	c	a	a	b	a	b

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.(MATh)

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	a	c	b	a	b	c	a	c	a	b	b	c	a	c	c

Question 16: Let $f(x) = \dfrac{3^x}{x}$ and $h = 0.1$ Compute the approximate value of $f''(3)$ and the absolute error. If $\max|f^{(4)}| = 6.1022$, then find the number of subintervals required to obtain the approximate value of $f''(3)$ within the accuracy 10^{-4}.

Solution. Given $x_1 = 3, h = 0.1$, then the formula for $f''(3)$ becomes

$$f''(3) \approx \frac{f(3+0.1) - 2f(3) + f(3-0.1)}{(0.1)^2} = D_h^2 f(3),$$

or

$$f''(3) \approx \frac{f(3.1) - 2f(3) + f(2.9)}{0.01} = \frac{3^{3.1}/3.1 - 2(3^3/3) + 3^{2.9}/2.9}{0.01} \approx 6.2755 = D_h^2 f(3).$$

To compute the absolute error for our approximation we have to compute the second derivative of $f(x)$ as follows:

$$f'(x) = (3^x[\ln(3)x - 1])/x^2,$$
$$f''(x) = (3^x[(\ln(3))^2 x^2 - 2\ln(3)x + 2])/x^3.$$

Since the exact value of $f''(1)$ is

$$f''(3) = (3^3[(\ln(3))^2(3)^2 - 2\ln(3)(3) + 2])/(3)^3 = 6.2709,$$

therefore, the absolute error $|E|$ can be computed as follows:

$$|E| = |f''(1) - D_h^2 f(1)| = |6.2709 - 6.2755| = 0.0046.$$

As the maximum value of the fourth derivative of the given function in the interval is given as

$$M = \max_{2.9 \le x \le 3.1} |f^{(4)}| = 6.1022,$$

and the accuracy required is 10^{-4}, so

$$|E_C(f,h)| \le \frac{h^2}{12} M \le 10^{-4},$$

we have (h=(3.1-2.9)/n)

$$\frac{(3.1-2.9)^2/n^2}{12} M \le 10^{-4}, \quad \text{solving for } n^2, \qquad n^2 \ge \frac{(6.1022)(3.1-2.9)^2(10^4)}{12},$$

by taking square root on both sides, we get, $n \ge 14.2621$, gives, $n = 15$. •

Question 17: Determine the number of subintervals required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using composite Simpson's rule. Then approximate the given integral and compute the absolute error.

Solution. We have to use the error formula of composite Simpson's rule which is

$$|E_{S_n}(f)| \leq \frac{(b-a)}{180} h^4 M \leq 10^{-4}.$$

Given the integrand is $f(x) = \frac{1}{x+4}$, and we have $f^{(4)}(x) = \frac{24}{(x+4)^5}$. The maximum value of $|f^{(4)}(x)|$ on the interval $[0, 2]$ is $\frac{3}{128}$, and thus $M = \frac{3}{128}$. Using the above error formula and $h = 2/n$, we get

$$\frac{48}{(90 \times 128 \times n^4)} \leq 10^{-4}, \quad \text{or} \quad n^4 \geq \frac{48 \times 10^4}{(90 \times 128)}.$$

Solving for n, gives

$$n \geq \left(\frac{48 \times 10^4}{(90 \times 128)} \right)^{1/4} = 2.5407,$$

so the number of even subintervals n required is $n = 4$. Thus the approximation of the given integral using $h = \frac{2-0}{4} = \frac{1}{2} = 0.5$ is

$$\int_0^2 \frac{1}{x+4} \approx \frac{0.5}{3} \Big[f(0) + 4[f(0.5) + f(1.5)] + 2f(1) + f(2) \Big],$$

$$\int_0^2 \frac{1}{x+4} \approx \frac{1}{6} \Big[0.25 + 4(0.2222 + 0.1818) + 2(0.2) + 0.1667 \Big] = 0.4055.$$

Since the exact value of the given integral is

$$\alpha = \int_0^2 \frac{1}{x+4} dx = \ln(1.5) = 0.4055,$$

so the absolute error is

$$AbsE = |exact\ solution - Approximate\ solution| = |0.4055 - 0.4055| = 0.0000,$$

up to 4 decimal places. •